Evaluation of Drawdown Curves Derived from Multiple Well Aquifer Tests in Heterogeneous Environments

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ABSTRACT

Aquifer coefficients derived from nonsteady-state, multiple well, aquifer tests in laterally heterogeneous environments often have uncertain meaning. Drawdown at observation wells reflects the removal of water from storage in the aquifer and transient refraction of ground water pathlines during the evolution of a non-symmetrical cone of depression. These effects are masked within observation well drawdown data such that "good" Theis (1935) type curve matches often result. Transmissivity and storativity values derived from independent drawdown curves plotted as drawdown versus time (t) or drawdown versus time/distance^2 (t/r^2) usually differ from observation well to observation well. These aquifer coefficients often are considered to represent some type of average of the materials between and/or about the pumping well and the observation wells. Simulations of two multiple well aquifer tests with simple, arbitrary distributions of block heterogeneities suggest that transmissivity (T) and storativity values derived from independent drawdown curves by the Theis (1935) method generally increase with distance from the pumping well. This apparent scale effect is related to the force-fitting of early-time drawdown data to the steep portion of the Theis type curve without sufficient late-time drawdown data to constrain vertical shifting of the drawdown data relative to the type curve.

Log-log plots of drawdown versus t/r^2 for multiple well aquifer tests form families of curves that are characteristic of the distribution of observation wells and the degree of heterogeneity within the cone of depression. Separation between discrete drawdown curves within a family provides a qualitative measure of the degree of heterogeneity within the cone of depression. All of the drawdown curves within a family converge on a single curve at large values of t/r^2. A composite analysis of all of the drawdown data within the family yields an estimate of the average T within the cone of depression. Analysis of discrete drawdown curves as integral members of the family of curves provides a means to constrain type curve matches and minimizes force-fitting if drawdown data are defined for large values of t/r^2 for at least one well. The constrained type curve matches provide more reasonable estimates for T near individual observation wells than analysis of drawdown curves independently.
KEYWORDS: Heterogeneity, aquifer testing, modeling, transmissivity, storativity, drawdown.

INTRODUCTION

The actual meaning of aquifer coefficients derived from drawdown curves during multiple well aquifer tests in heterogeneous aquifers is typically unknown. Unlike slug tests, which measure the hydraulic conductivity of a very small volume of aquifer in the immediate vicinity of the well bore, multiple-well aquifer tests yield integrated drawdown curves for much larger, undefined, volumes of the aquifer. These integrated drawdown curves yield values for aquifer coefficients that have uncertain meaning with respect to the actual properties of the aquifer stressed during an aquifer test. The uncertainty typically manifests itself as variable, calculated T and S values for different observation wells when the Theis (1935) or Cooper and Jacob (1946) methods are used to analyze drawdown data. Bibby (1979) suggested that no physical justification exists for the use of these formulas in heterogeneous environments and that T and S values derived in these environments are simply numbers with no more meaning than the slope of the drawdown curves from which they are derived. The uncertainty is increased when one considers whether drawdown curves developed for the relatively small number of observation points (often < 10 wells) used in most tests, adequately represent some type of localized average aquifer response to heterogeneities within the cone of depression formed by the pumping well. This uncertainty is often attributed to a lack of *a priori* information on the distribution of heterogeneities in the subsurface, combined with the limitations of currently available analytical methods for aquifer test analysis. The subjective nature of aquifer test analysis generally contributes relatively little to overall uncertainty if "good" type curve matches or straight line plots result. Derived aquifer coefficients often are suggested to represent aquifer materials near each well or some type of average of the materials between the pumping well and the observation well.

Nonequilibrium methods of aquifer test analysis for confined aquifers, i.e., Theis (1935) and Cooper and Jacob (1946) constitute the initial choice of analysis for most aquifer tests. Usually other methods of analysis are not considered unless log-log plots of drawdown data versus time (t) or time/distance^2 (t/r^2) deviate from the "Theis" type curve or a semi-log (Cooper and Jacob, 1946) plot of the data deviates from a straight line. The representativeness of aquifer coefficients derived from "good" Theis type curve matches or straight line plots of observation well drawdown data is discussed in this paper. Drawdown curves produced by simulated aquifer tests in laterally heterogeneous environments with known distributions of heterogeneities are evaluated. Factors that affect the interpretation of plotted drawdown curves also are described.

Previous Investigations

Much research has been conducted on the development of methods to estimate the hydraulic properties of heterogeneous porous media. Many researchers have focused on stochastic descriptions of synthetic heterogeneous media to define "average", "effective", or "equivalent" hydraulic properties. Comparatively few researchers have focused on the plight of the practicing ground water hydrologist who must interpret real aquifer drawdown responses for site characterization or regulatory purposes. Cardwell and Parsons (1945) suggested that the equivalent transmissive capacity of randomly distributed block heterogeneities lies between the harmonic and arithmetic means of the actual transmissive capacities of the heterogeneities. Warren and Price (1961) simulated flow in porous media composed of random heterogeneities and concluded that the geometric mean of the heterogeneities represented a good estimate of the effective permeability. Toth (1966) presented one of the first published accounts of the use of log-log and semi-log plots of late-time drawdown data (field data) to evaluate long-term aquifer yield for a heterogeneous aquifer. He showed that log-log and semi-log plots of late-time drawdown data for multiple observation wells converged on single curves or straight lines, respectively, which represented large-scale average conditions. Toth (1966) showed that early-time drawdown data generally produced higher and much more variable estimates for T than late-time data. He noted also that a wide range of calculated S values was produced during multiple-well aquifer tests. Freeze (1975) furthered the work of Warren and Price (1961) and questioned whether aquifer coefficients determined by aquifer testing are representative of the stochastic properties of a non-uniform formation. Vandenberg (1977) directed his investigation toward the practicing ground water hydrologist when he duplicated and extended the work of Warren and Price (1961); he found average T values for the model nodes to be closer to the arithmetic mean than the geometric mean. Bibby (1977) evaluated 122 drawdown curves for pumping wells in heterogeneous, clastic sediments. He described four basic drawdown curve shapes and used the Cooper and Jacob (1946) method to ascertain the "short-term transmissive capacity" near each well. Bibby (1979) discussed the meaning of aquifer coefficients derived from early and late-time data with respect to the estimation of local and regional averages. He defined weighted arithmetic, harmonic and geometric means for the long-term transmissive capacity of a drainage volume divided into concentric rings centered on the pumping well. Barker and Herbert (1982) simulated an aquifer test conducted in a "patchy" aquifer to aid interpretation of the aquifer drawdown response. Streltsova (1988) suggested that calculated T and S values are averages of the between-well properties and the properties surrounding the wells; however, heterogeneities near the pumping well were suggested to exert much more influence on the drawdown response than those located farther from the well. Streltsova (1988) also suggested that the type of averaging that occurs depends on the areal distribution of the heterogeneities. Butler (1986, 1988, 1990) and Butler and McElwee (1990) presented insights into the interpretation of aquifer response data for aquifers composed of radially-symmetrical, disk-shaped heterogeneities. According to Butler (1988), the Theis (1935) method will yield estimates of T and S that are weighted averages of near-well and far-field properties for heterogeneities that are distributed radially symmetrical about the pumping well. Butler (1991) extended his earlier work by incorporating the effects of lateral heterogeneity into his analysis of aquifer drawdown response. He concluded that T values derived from observation well drawdown curves provide reasonable estimates for most practical applications. Butler and Liu (1993) analyzed the effects of the presence of a disk-shaped heterogeneity on observation well drawdown. The effects were evaluated for different radial and angular locations from the pumping well. They showed that measurable effects on observation well drawdown were primarily a function of the distance between the disk and the pumping well. Butler and Liu (1993) concluded that constant rate aquifer tests are not very effective for the characterization of lateral variations in flow properties. Schad and Teutsch (1994) compared the T and S values derived from several small-scale and large-scale aquifer tests in a braided stream environment. They suggested that the effective length scale of the heterogeneity structure, but not true effective T and S values, could be estimated from their aquifer test data. Meier et al. (1998) suggested that the straight-line method (Cooper and Jacob, 1946) will provide a good approximation of the effective T in multilognormal and non-multilognormal T fields when constrained to late-time data. However, like Toth (1966), they found that estimates of S ranged widely.

Desbarats (1987) used a numerical approach to estimate effective permeability of sand-shale formations under steady state flow conditions. He suggested that effective permeability in his investigation was dependent upon the shale volume fraction, the spatial co-variance structure, and the dimensionality of the flow system. Naff (1991) used a perturbation solution for three-dimensional radial flow in heterogeneous porous media. He concluded that the effective hydraulic conductivity will be essentially constant at a distance greater than two to three length scales from the well bore and will have a value dependent upon the statistical anisotropy of the medium. Desbarats (1992) investigated steady state flow in a heterogeneous medium using a combined numerical-empirical approach. He found that the effective transmissivity of a single-well radial system could be estimated as a spatial geometric average of point transmissivities weighted by their inverse squared distance from the well bore axis. Desbarats (1993) extended his investigation to steady state flow between an injection well and a pumping well. He concluded that inter-well transmissivity could be represented by the harmonic mean of transmissivities averaged over circular regions centered at each well. Dykaar and Kitanidis (1992) used a numerical spectral method to compute the effective hydraulic conductivity of two and three-dimensional, isotropic, stationary, log-normal hydraulic conductivity distributions. They evaluated different averaging volumes within a random field and found that about 80 integral scales were needed for effective hydraulic conductivity to approach an asymptotic value. Dykaar and Kitanidis (1993) extended their work to an aquifer of variable thickness and found that an averaging volume of about 10 horizontal integral scales was required to obtain a value for effective transmissivity. Dykaar and Kitanidis (1993) suggested that different transmissivities in nearby wells were due to small-scale variations in the hydraulic conductivity around the wells where the measurements were taken. Oliver (1993) used a perturbation approach to evaluate the effects of two-dimensional areal variations in T and S on observation well drawdown. He suggested that the area of influence of observation well drawdown is bounded by an ellipse that encloses the pumping well and the observation well.

Theoretical Considerations

Ground water produced by a well in an extensive confined aquifer is derived from storage within the aquifer. Discharge (Q) from the well must be equal to the product of the aquifer storativity (S) and the rate of decline in head (h) integrated over the area affected by pumping (Davis and DeWiest, 1966)

$$dQ = -Srd\theta dr\frac{\partial h}{\partial t} \quad \text{and} \tag{1}$$

$$Q(t) = -S\int_{r_w}^{\infty}\int_{0}^{2\pi} r\frac{\partial h(r,\theta,t)}{\partial t}d\theta dr \tag{2}$$

where: r is distance from the pumping well, L; θ is the angular direction, rad; and t is time.

If the extensive aquifer is homogeneous and isotropic, the distribution of head about the pumping well is radially symmetrical and

$$Q = -2\pi S\int_{r_w}^{\infty} r\frac{\partial h(r,t)}{\partial t}dr \tag{3}$$

If it is assumed that the solution depends only on the distance r, the radial coordinate form of the diffusion equation for flow to a well in a confined aquifer can be written as (de Marsily, 1986)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)=\frac{S}{T}\frac{\partial h}{\partial t} \quad 4$$

Theis (1935) developed a solution to Eq.4 for the following initial and boundary conditions (Domenico and Schwartz, 1990):

$$h(r,0)=h_0$$

$$h(\infty,t)=h_0$$

$$\lim_{r\to 0}\left(r\frac{\partial h}{\partial r}\right)=\frac{Q}{2\pi T} \qquad \text{for } t>0$$

The Theis (1935) solution is

$$h_0-h=s=\frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-z}}{z}dz \quad 5$$

where: u equals $\frac{r^2S}{4Tt}$; h_0 is the original head at any distance r from a fully penetrating well at time t equals zero, L; s is the drawdown due to pumping, L; Q is the constant pumping rate, L^3/T; T is the aquifer transmissivity, L^2/T; and S is the aquifer storativity, dimensionless.

Application of Eq.5 to heterogeneous aquifers is problematic because it no longer can be considered to describe drawdown due to pumping uniquely in any radial direction from a pumping well. Drawdown generally is not radially symmetrical in heterogeneous environments because the refracted pathlines that fluid particles follow in route to the pumping well change with position as the cone of depression grows.

Volumetric Nature of a Cone of Depression

The volume of a cone of depression in the potentiometric surface of a confined aquifer is defined uniquely in an aquifer of constant storativity by:

$$V=\frac{Q}{S}t \quad 6$$

where: V is the volume of the cone of depression, L^3; Q is the constant pumping rate, L^3/T; S is the storativity, dimensionless; and t is the time since pumping began.

While the S controls the volume of the cone of depression for a given volume of pumping, T exerts a greater control over the ultimate shape and extent of the cone. The value of u in Eq. 5 depends on time, distance, T and S; and u determines the radius of the cone of depression (Theis, 1940; Domenico, 1972). This fact is well known. However, in a heterogeneous aquifer, S and T as well as the volumetric rate of growth of a cone of depression may vary in space. Therefore the S, T and physical volume of an individual heterogeneity control the "volume portion" of the total cone of depression that is due to that heterogeneity. A volume portion is defined in this paper as the percentage of the total volume of a cone of depression that exists within an individual heterogeneity of constant T and S. Use of Eq.5 involves matching a real, observation well drawdown response which incorporates spatially and temporally variable effects of pathline refraction, with an ideal theoretical response (i.e., type curve).

If an aquifer is homogeneous and isotropic, a log-log plot of drawdown data versus t/r^2 for all observation wells will fall on a single curve which reveals the exact profile of the cone of depression as a function of distance r^2 from the pumping well (Domenico, 1972). However, in a heterogeneous aquifer, a plot of drawdown versus t/r^2 for multiple observation wells will produce a family of curves rather than a single profile of the cone of depression. The family of curves will represent a measure of the spatial distribution of drawdown about the mean condition to the degree to which the family is defined by a limited number and distribution of observation wells. Toth (1966) recognized that drawdown curves for multiple observation wells in a heterogeneous aquifer tended to converge on a single curve at later times as the cone of depression stabilizes. He noted that "apparent transmissibility" seemed to decrease with increasing length of pumping.

Drawdown stabilizes as a function of radial distance (r) from the pumping well and time (t). Therefore, a family of drawdown curves will tend to converge at large values of t/r^2. At small values of t/r^2 separation between individual drawdown curves, provides a qualitative measure of the degree of heterogeneity within the cone of depression. Prior to drawdown curve convergence, the Theis (1935) method or the Cooper and Jacob (1946) method will yield different calculated T and S values for each drawdown curve. Bibby (1979) suggested that the only meaningful parameters produced under these conditions are the drawdown curves themselves. He noted that the drawdown curve is the signature of the well and is different for every well. Bibby (1979) suggested also that the early-time portion of a drawdown curve probably reflects the average or effective local conditions near the well while the late-time portion of a drawdown curve reflects more regional average or effective conditions. Bibby (1979) and Butler (1988) suggested that the late portions of drawdown curves can be used to estimate sustainable aquifer yield.

The purpose of this paper is to illustrate some of the major factors that affect the shape of drawdown curves, and the interpretation of aquifer coefficients derived from those curves, in laterally heterogeneous aquifers. The objective is to evaluate drawdown curves in environments that, while not geologically realistic in form, are relatively easy to visualize and interpret. Aquifer tests in confined aquifers with different, known, hypothetical distributions of heterogeneities were simulated with MODFLOW (McDonald and Harbaugh, 1988). The simulated drawdown data were analyzed individually by the Theis (1935) method to evaluate the magnitude of calculated, aquifer coefficients relative to arbitrary distributions of known values. The Cooper and Jacob (1946) method was not used to analyze the drawdown data because, for most observation wells, the critical time (t_c) for $u \leq 0.01$ was not satisfied.

MODELING CONDITIONS

MODFLOW was used to simulate two-dimensional, transient ground water flow to a pumping well in a confined aquifer with simple, lateral heterogeneities. Two models were developed and run using Visual MODFLOW© (Waterloo Hydrogeologic, Inc.). The zonation approach was used whereby the region of ground water flow for each simulation was divided into distinct zones (block heterogeneities) with constant T and S values assigned to each zone. The distribution of heterogeneities was chosen so that calculation of the volumetric distribution of the cone of depression within the heterogeneities would be a tractable problem. In addition, the distribution was chosen to illustrate clearly that aquifer coefficients derived from independent drawdown curves may not reflect actual values or meaningful average values near the wells.

Each two-dimensional model consisted of identical, 249 x 249 finite-difference grids with 10 m uniform grid spacings ($\Delta x, \Delta y$). Two, multiple observation well, aquifer tests (Aquifer Test 1 and Aquifer Test 2) in a confined aquifer with a constant thickness of 30 m and a uniform storativity (S) of 0.005 were simulated. Impermeable boundaries were placed around the perimeter of the model grids beyond the influence of pumping and a single, fully penetrating, pumping well with a constant discharge of 545 m^3/d was located at the center of each grid at node (125, 125). Model input values for T and S, and the pumping rate were chosen to control the rate of growth of the cone of depression with respect to the boundary locations. The initial drawdown everywhere in the aquifer was zero for all simulations. Each aquifer test consisted of a 24-hour stress period divided into 25 time steps with a time-step multiplier of 1.2.

Osiensky and Williams (1997) showed that MODFLOW produces very accurate results for the pumping and boundary conditions described above with use of the PCG2 method for matrix solution. The PCG2 method for matrix solution was used for all simulations in this paper. MODFLOW-generated drawdown values for the grid spacings and boundary locations used for the simulations were evaluated for accuracy. T and S values were calculated for both near and distant observation well (model node) locations for two, 24-hour aquifer test simulations with homogeneous and isotropic conditions. Values for T used in these tests were 60 m^2/d and 300 m^2/d, respectively. The value for S used in both tests was 0.005. These T values incorporated the full range of T values used for specific block heterogeneities in the simulations of aquifer tests 1 and 2. AquiferTest©(Waterloo Hydrogeologic, Inc.) software was used to analyze drawdown data versus time by manually matching to a Theis (1935) type curve. T and S values derived for several randomly selected observation well locations were identical to the values used as input for the two test simulations. Drawdown along the perimeter of the model grids was $<$ 0.0001 m for the duration of all simulated aquifer tests.

Aquifer Test 1

The model grid for aquifer test 1 was divided into 16 separate zones with 9 different T values (Figure 1). The pumping well was located at the geometric center of the model grid within the upper left hand corner of Zone 11 (T = 60 m^2/d).

Simulation of aquifer test 1 was designed specifically to evaluate the volumetric effects that each of the 16 heterogeneities exerted on the growth of the cone of depression over time and the resulting drawdown curves. The volumetric rate of growth of specific portions of the cone of depression was a function of the pumping rate, and the hydraulic diffusivity (T/S) of each heterogeneity contacted by the entire cone at a particular point in time. To evaluate these effects during aquifer test 1, the percentage of the volume of the cone of depression contained within each of the 16 heterogeneities was estimated for the 25 time steps of the simulation. Volumes were estimated by the trapezoidal rule with SURFER®(Golden Software, 1995). The estimated volumes were used to derive arithmetic, harmonic and geometric weighted mean T values for all of the aquifer materials contacted by the cone of depression for each time step as follows (Bibby, 1979):

$$AWMT = \frac{\sum_{i=1}^{n} w_i T_{H_i}}{\sum_{i=1}^{n} w_i} \qquad 7$$

$$HWMT = \frac{1}{\frac{\sum_{i=1}^{n} \frac{1}{T_{H_i}} w_i}{\sum_{i=1}^{n} w_i}} \qquad 8$$

$$GWMT = \left(\prod_{i=1}^{n} T_{H_i}^{w_i} \right)^{\frac{1}{\sum_{i=1}^{n} w_i}} \qquad 9$$

where: $AWMT$ is the arithmetic weighted mean T, L^2/T; $HWMT$ is the harmonic weighted mean T, L^2/T; $GWMT$ is the geometric weighted mean T, L^2/T; w_i is the weighting factor equal to (V_{H_i} / V_T); V_{H_i} is the volume of the cone of depression within the i^{th} heterogeneity at time t, L^3; V_T is the total volume of the cone of depression at time t, L^3; and T_{H_i} is the actual T of the i^{th} heterogeneity, L^2/T.

The boundary (i.e., total lateral extent) of the cone of depression for each time step, and after 24 hours of pumping, for both aquifer test simulations was assumed to be represented by a drawdown of 0.0001 meters. Most of the cone of depression was contained within zone 11 for the first few minutes of the aquifer test 1 because of the pumping well location in the upper left corner of this zone. However, cone growth was not radially symmetrical and at later times the cone of depression grew preferentially in the higher T/S zones at the expense of further cone expansion in zone 11. This resulted in continually increasing AWMT, HWMT and GWMT values (Figure 2) with time as higher T/S materials exerted greater and greater influence over the spatially varying, volumetric growth rate of the cone of depression. The weighted mean T values increased most rapidly for early values of time (<600 minutes) when the cone of depression was being established in the aquifer materials close to the pumping well. This finding is consistent with the work of Streltsova (1988), Butler (1988,1991) and Butler and Liu (1993), who indicated that the aquifer properties close to the pumping well exert a much greater influence on observation well drawdown than aquifer materials at other locations in the aquifer. At later times (>600 minutes), the AWMT, HWMT and GWMT values approached constant values as the cone of depression developed toward a steady shape. The effects of spatial variation in T propagated throughout the entire cone of depression as a function of time.

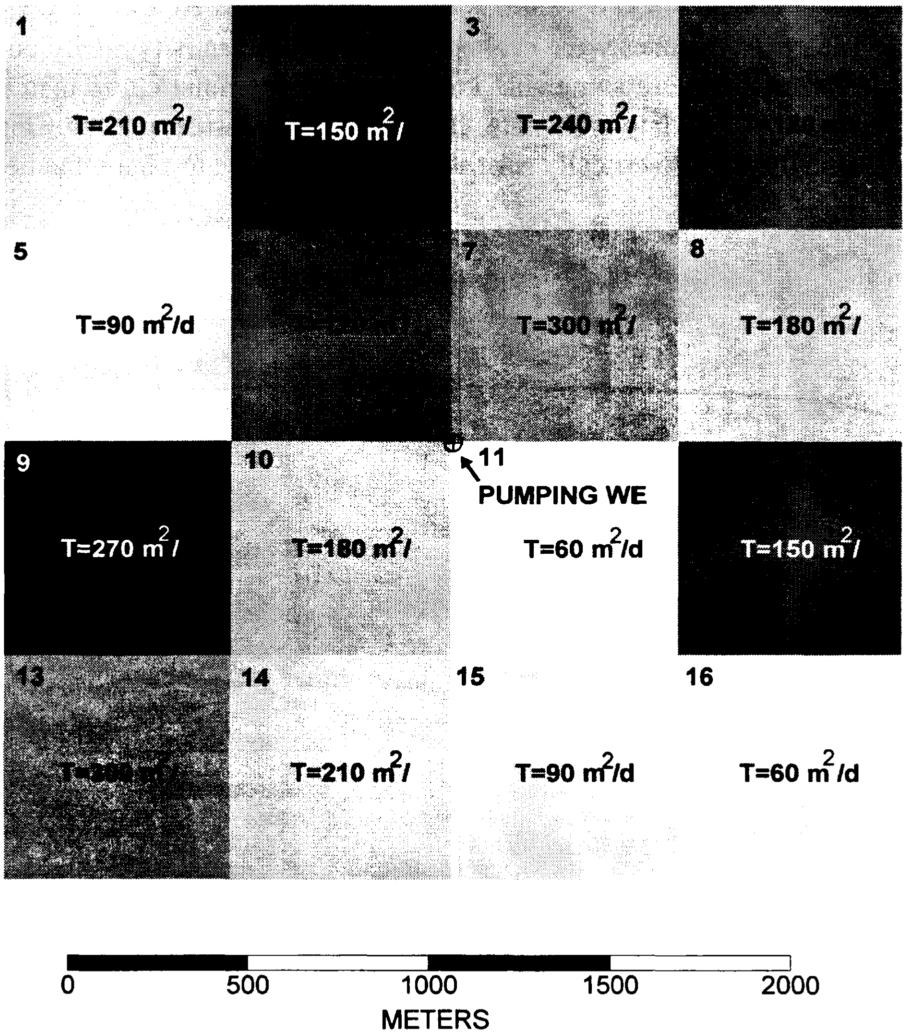

Figure 1. Hydrogeologic map of the 16 zones of varying transmissivity for aquifer test

Three sets of stratified, random, observation well locations (nodes) were selected to evaluate the spatial relationships of T and S values derived from observation well drawdown data. The three sets of observation well locations were selected by dividing the area of the cone of depression (i.e., total measurable drawdown $\geq$ 0.005m) formed during the 24-hour aquifer test into 10, 21 and 37 equal sized blocks, respectively. Total drawdown of 0.005m at the observation wells after 24 hours of pumping was considered the minimum amount needed for analysis. The coordinates for each well within a block (one well per block) were selected randomly. All 68 of the observation wells experienced sufficient drawdown for analysis during an aquifer test; however, 6 of the 68 observation wells (wells 1,2,7,41 and 60) experienced less than 0.005m of drawdown during aquifer test 2. Figure 3 shows the locations of the 68 observation wells. Independent log-log plots of drawdown versus t/r^2 were developed and analyzed for each observation well. T and S values for the three sets of wells (total of 68 wells) were solved with Aqtesolv™ (Duffield, 1996); the type curve matches and T and S values were confirmed with AquiferTest©(Waterloo Hydrogeologic, Inc.). The curve matches were evaluated subjectively, as is typically done when details of the geological conditions are somewhat known. However, most curve matches were not constrained by both early and late-time data. This allowed considerable shifting of the drawdown curves horizontally and/or vertically relative to the type curve to obtain the best match curve.

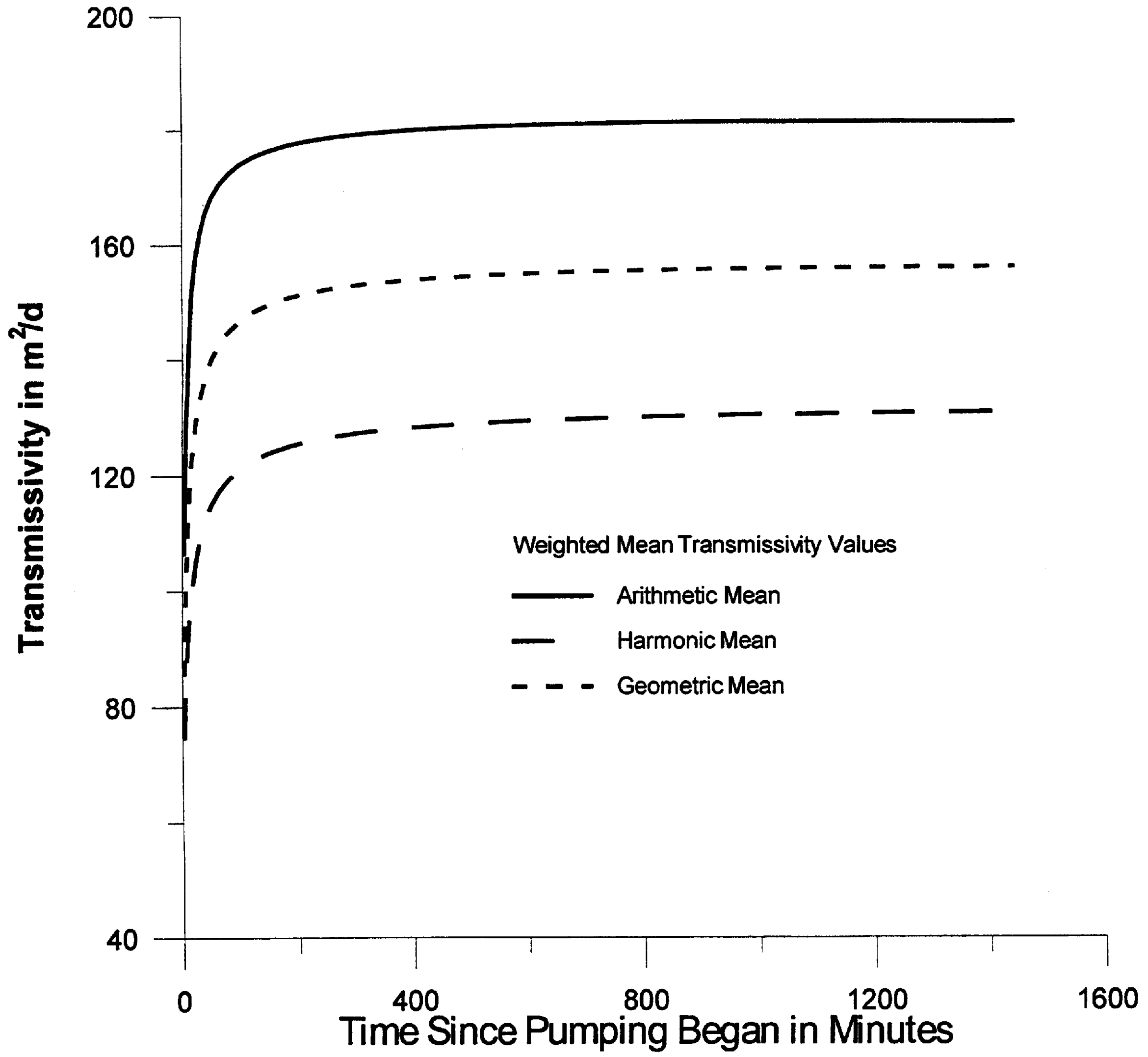

Figure 2. Arithmetic, harmonic and geometric weighted mean transmissivity values within the cone of depression of aquifer test 1.

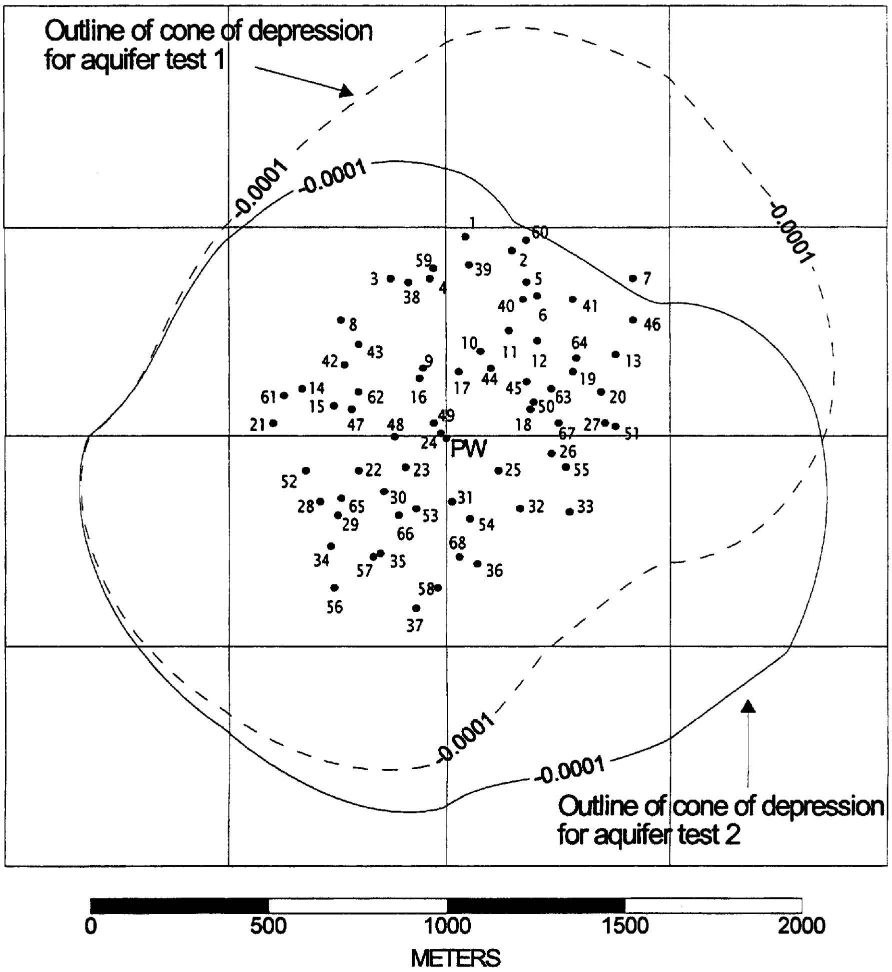

Figure 3. Cones of depression formed during aquifer tests 1 and 2. The dashed line represents 0.000 meters of drawdown after 24 hours of pumping for aquifer test 1. The solid line represents 0.0001 meters of drawdown after 24 hours of pumping for aquifer test 2.

Figure 4 presents an example log-log plot of independent, Theis type curve matches for observation wells 24, 32 and 35 at distances of 22.36 m, 290m, and 375.9m, respectively, from the pumping well. The number of drawdown measurements plotted (i.e., measurable drawdown ≥ 0.001m) for each observation well was a function of time, distance from the pumping well, and the position of the observation well. Five data points were the minimum number of drawdown measurements used for analysis. Observation wells located close (many drawdown measurements) to the pumping well generally yielded lower T values than observation wells located far (few drawdown measurements) from the pumping well regardless of the actual T near the observation well (Figure 5). The relationship between calculated S and observation well distance from the pumping well was not as clear (Figure 6). Storativity was overestimated in approximately 66% of the observation wells 6). Notable was the apparent lack of a definable relationship between the T and S calculated for specific observation wells and and the actual T and S values near the wells. For example, all observation wells in zone 11 yielded T values at least two times greater than the actual T (60 m^2/d) in this zone. S was overestimated for all wells in zone 11. This suggests that the drawdown curves reflected some type of average of the heterogeneities contacted by the entire cone of depression rather than average conditions near the observation wells. Observation wells located beyond a distance of about 550 meters from the pumping well yielded higher T values (>300 m^2/d) than actually existed in any of the 16 zones. These T values clearly were not definable "averages" of the T values within the flow domain. However, the drawdown curves for distances beyond 550 meters were defined only for early-time data (i.e., small t/r^2) prior to stabilization of the cone of depression.

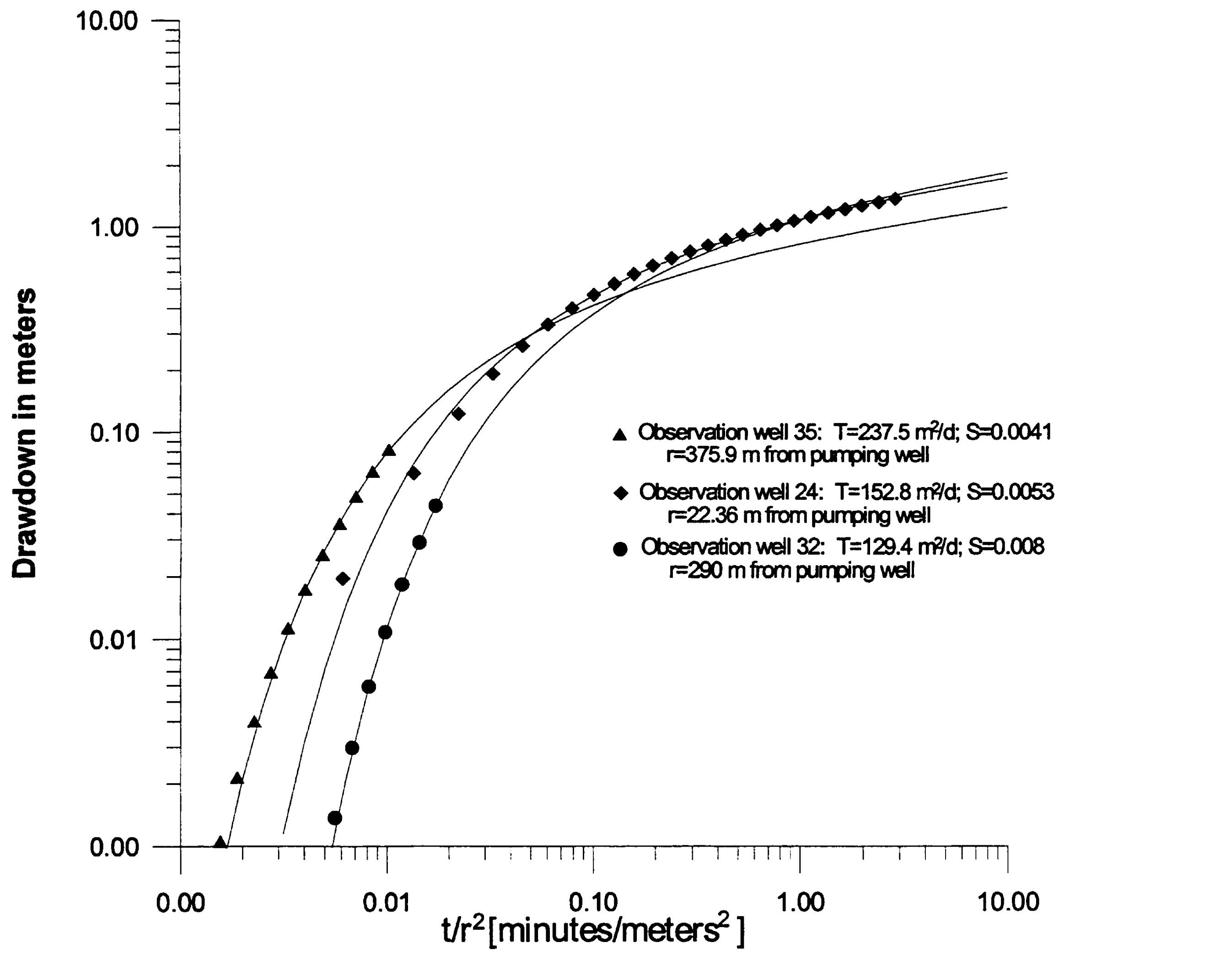

Figure 4. Representative, independent Theis type curve matches for three of the 68 observation wells for aquifer test 1. Theis curve matches were produced by AQTESOLV using nonlinear weighted least-squares parameter estimation.

The volumetric rate of growth of the cone of depression was related directly to the uniform S of the aquifer for aquifer test 1. Thus, distortion of the cone of depression (Figure 3) was a function of the T distribution only. However, distortion of the cone of depression was masked within the drawdown data for individual observation wells. McElwee and Yukler (1978), and Serrano

(1997) showed that drawdown predicted by the Theis equation is more sensitive to T than to S near the pumping well, and that the sensitivity decreases with distance from the pumping well. McElwee and Yukler (1978) also showed that the effect due to a change in S is significant over a larger area than that due to a change in T. However, the specific effects of spatial variability in T and/or in S on measured drawdown data generally are not discernible during the graphical curve matching procedure. Therefore, indeterminate T and S values are derived (rather than unique values) based on the comparison between a plot of actual drawdown data that reflect the net effects of heterogeneities and an ideal drawdown curve (type curve).

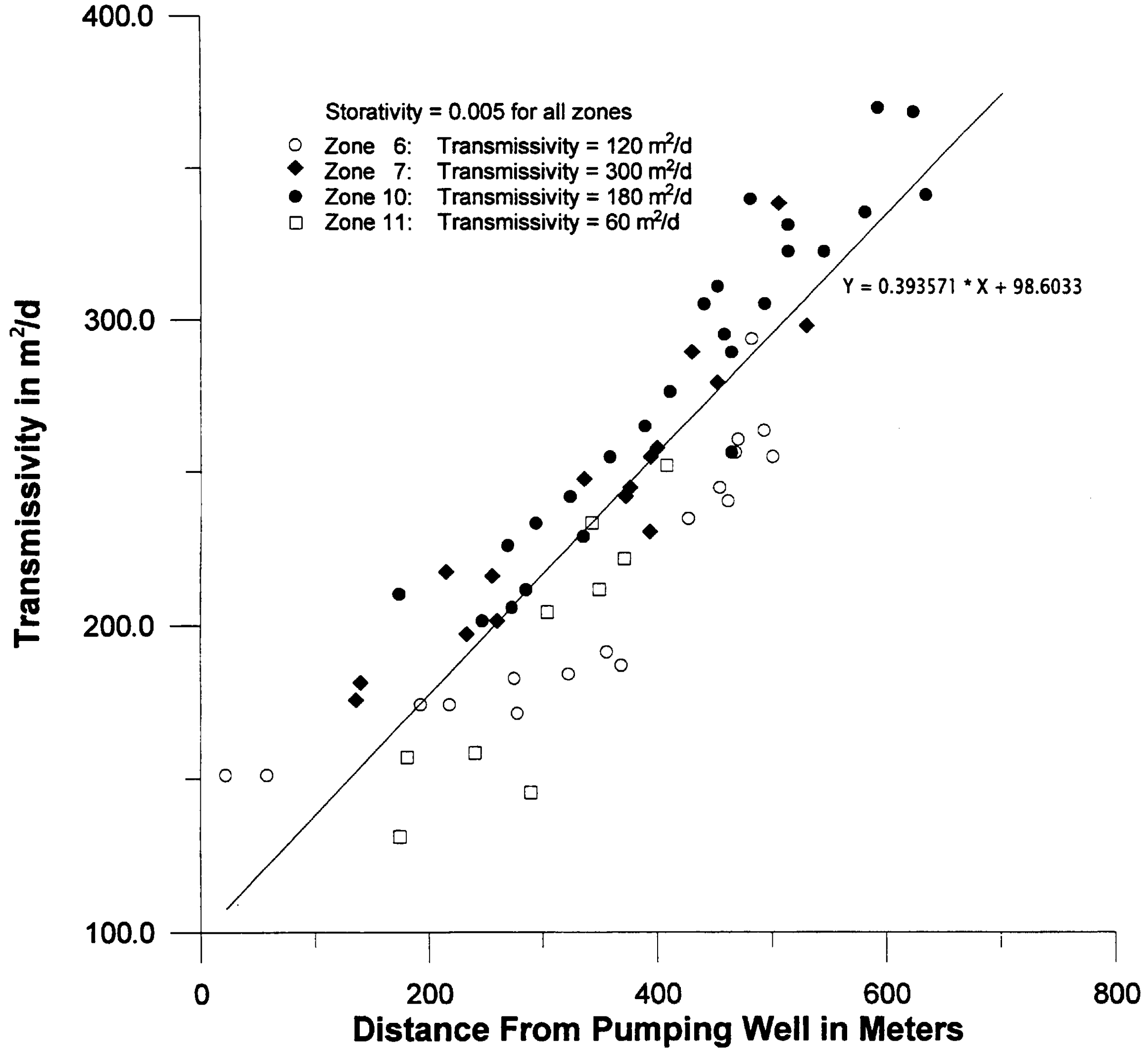

Figure 5. Plot of transmissivity values for the 68 observation wells versus distance from the pumping well for aquifer test 1.

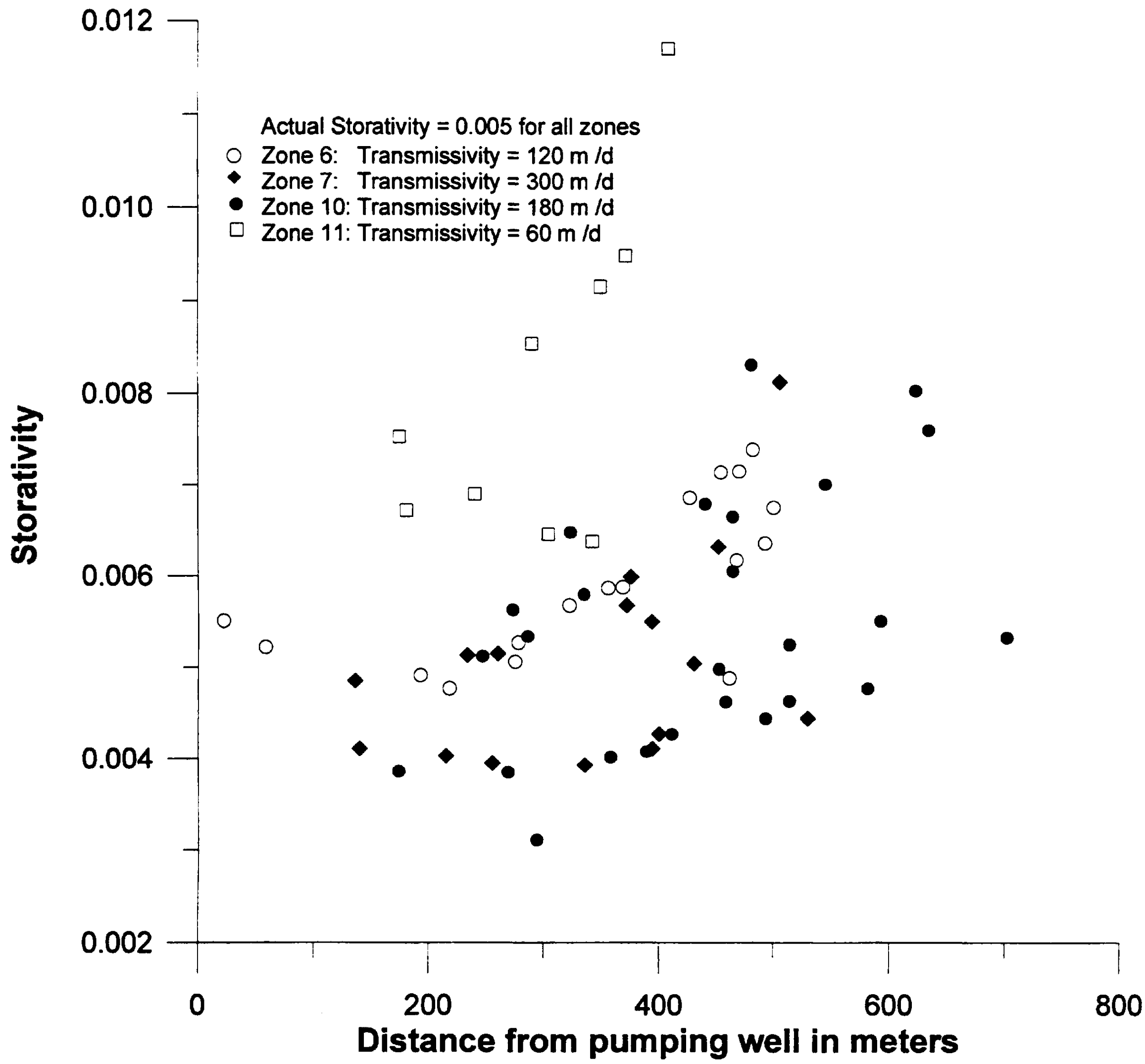

Figure 6. Plot of calculated storativity values for the 68 observation wells versus distance from the pumping well for aquifer test 1.

Aquifer Test 2

For aquifer test 2, the material properties for zones 7 and 11 were interchanged. Zone 11 was replaced with T=300 m^2/d and zone 7 was replaced with T=60 m^2/d. The material properties of all other zones were the same as for aquifer test 1 (Figure 7). Simulation of aquifer test 2 was designed to evaluate the effects that placement of the pumping well in high T material would have on the volumetric rate of growth of the cone of depression over time and the resulting drawdown curves. The percentage of the total volume of the cone of depression contained within each of the 16 heterogeneities was calculated for the 25 time steps of the simulation in the same manner as aquifer test 1. Time varying, AWMT, HWMT and GWMT values were determined based on Eqs. 7, 8, and 9, respectively (Figure 8). Comparison of Figures 2 and 8 suggests that plots of the AWMT, HWMT and GWMT values for both aquifer tests eventually would converge on single values that represent the regional arithmetic, harmonic and geometric mean T values, respectively. However, spatial differences in the volumetric growth rate of the cone of depression for aquifer test 2 relative to aquifer test 1 resulted in significantly different cone shapes (Figure 3). To allow direct comparison of T and S values derived for aquifer test 2 with those derived for aquifer test 1, drawdown data for 62 of the 68 observation wells, as described earlier, were analyzed. Drawdown curves for aquifer tests 1 and 2 generally yielded different values for T and S. However, Figures 9 and 10 show that plots of calculated T and S, respectively, for the 62 observation wells follow the same general trends of increasing T and S with distance from the pumping well. Figure 11 presents example Theis type curve matches for observation wells 24, 32, and 35 for aquifer test 2 (compare to Figure 4).

Analysis of Aquifer Test Data

Analysis of the drawdown curves independently prior to stabilization of the cone of depression is problematic because drawdown values for large t/r^2 are not available to constrain a type curve match. This leads to force-fitting of early-time data, which incorporate the transient effects of pathline refraction, to the steep portion of the type curve without good vertical control over the fit. Vertical control over the fit of drawdown data to a type curve generally decreases with distance from the pumping well and is a function of the length of pumping. The result is that T (Figures 5 and 9) and S (Figures 6 and 10) values estimated from independent drawdown curves appear to increase with distance from the pumping well.

Analysis of multiple drawdown curves as a family provides a means to constrain type curve matches and minimizes force-fitting if drawdown data are defined for large values of t/r^2 for at least one well. The shape of the family of curves for multiple observation wells is characteristic of the distribution of observation wells and the degree of heterogeneity within the cone of depression. All drawdown curves tend to converge on a single curve that best defines drawdown for large values of t/r^2 (i.e., the nearest observation well to the pumping well). Figures 12 and 13 show the family of drawdown curves produced for aquifer tests 1 and 2, respectively. Each figure represents a plot of all drawdown values ≥ 0.001m for the observation wells used in the analyses. A single Theis type curve was fitted to each family of drawdown curves using the nonlinear, weighted, least-squares parameter estimation technique in Aqtesolv™ (Duffield, 1996). Based on these curve matches, "average" T and S values were calculated for aquifer tests 1 and 2. Aquifer test 1 produced T=158.1 m^2/d and S=0.005. Aquifer test 2 produced T=173.5 m^2/d and S=0.0057. The average T values derived from the family of curves for aquifer test 1 and aquifer test 2 were most consistent with the GWMT values estimated after 24 hours of pumping (156.2 m^2/d and 165.2 m^2/d, respectively). Figures 12 and 13 show that both aquifer tests produced similar characteristic families of drawdown curves. However, drawdown curves for the same observation wells plotted at somewhat different locations (i.e., shifted left or right) for aquifer tests 1 and 2.

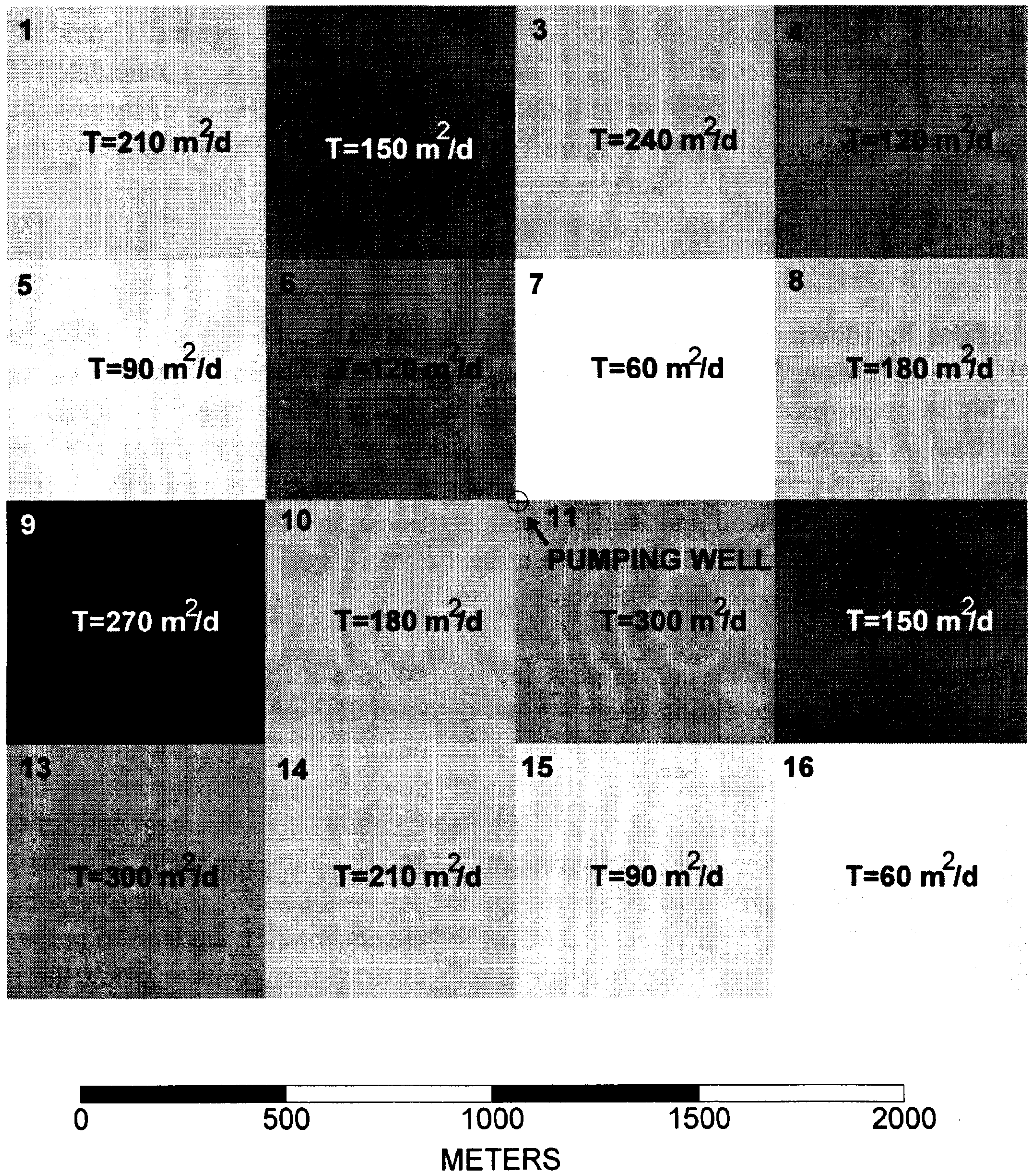

Figure 7. Hydrogeologic map of the 16 zones of varying transmissivity foaqfe2

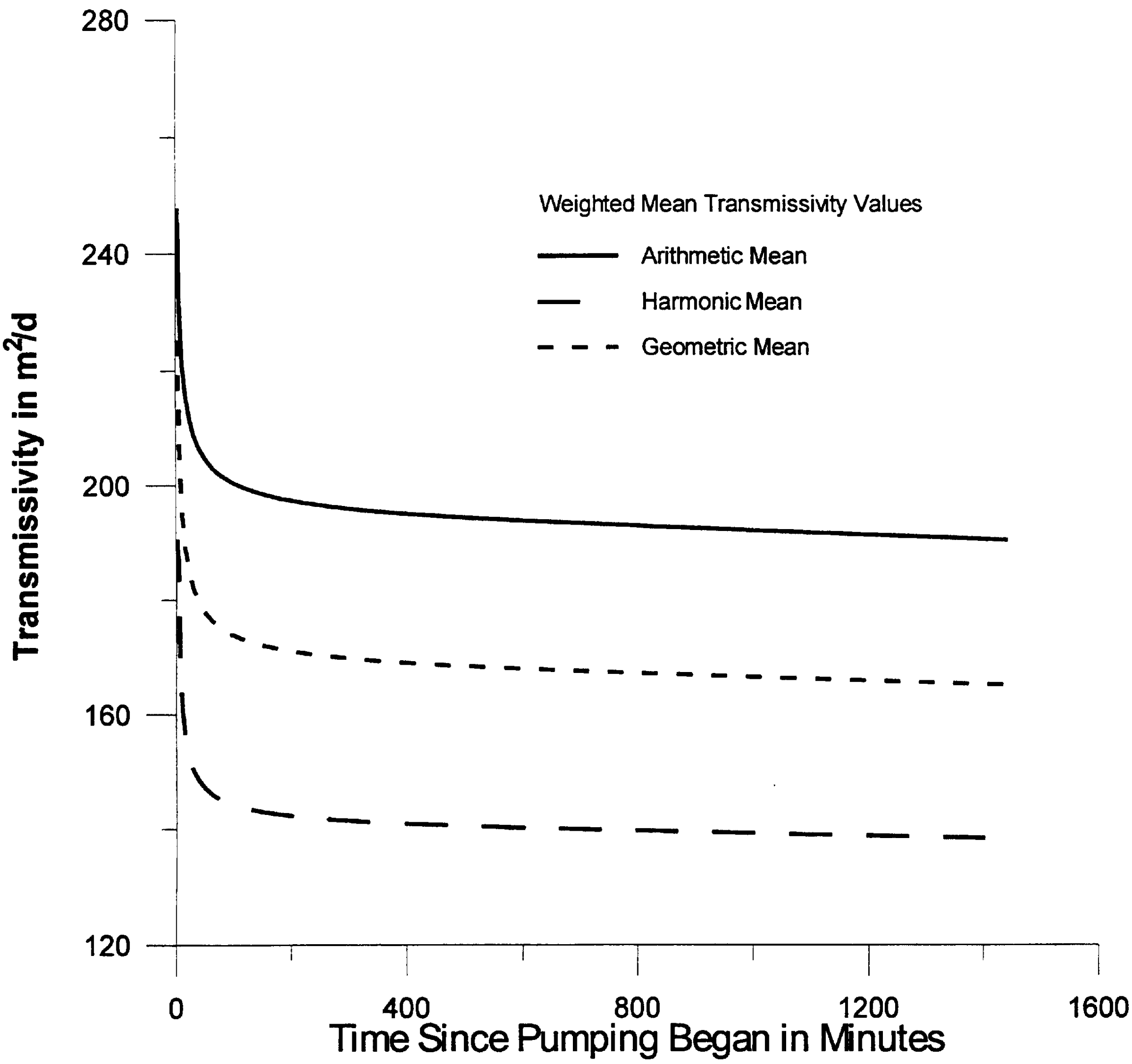

Figure 8. Arithmetic, harmonic and geometric weighted mean transmissivity values within the cone of depression of aquifer test 2.

Analysis of discrete drawdown curves as integral members of a family of curves for each aquifer test yielded different ranges for T and S than analysis of the drawdown curves independently. Drawdown data for large values of t/r^2 in the family of curves served to constrain vertical positioning of the type curve relative to the drawdown data during the matching procedure. Figure 14 presents an example of a constrained type curve match for aquifer test 2. The result was that more reasonable ranges of "average" T values were derived compared to analysis of the same drawdown curves independently. The family of 68 drawdown curves for aquifer test 1 produced T values ranging from T=139.1 to T=179.7 m^2/d; independent analysis of the same 68 drawdown curves produced T values ranging from T=131.0 to T= 370.1 m^2/d. The family of 62 drawdown curves for aquifer test 2 produced T values ranging from T=145.0 to T=197.1 m^2/d. Independent analysis of those 62 drawdown curves produced T values ranging from T=137.1 to

T=306.7 m²/d. Estimates for S were affected by the vertical and horizontal positions of the type curve matches for both the independent analyses and the family analyses. Therefore, the S values derived from the drawdown curves reflect the spatial variations in T and the resulting distortions in the cone of depression rather than average values for the aquifer within the cone of depression.

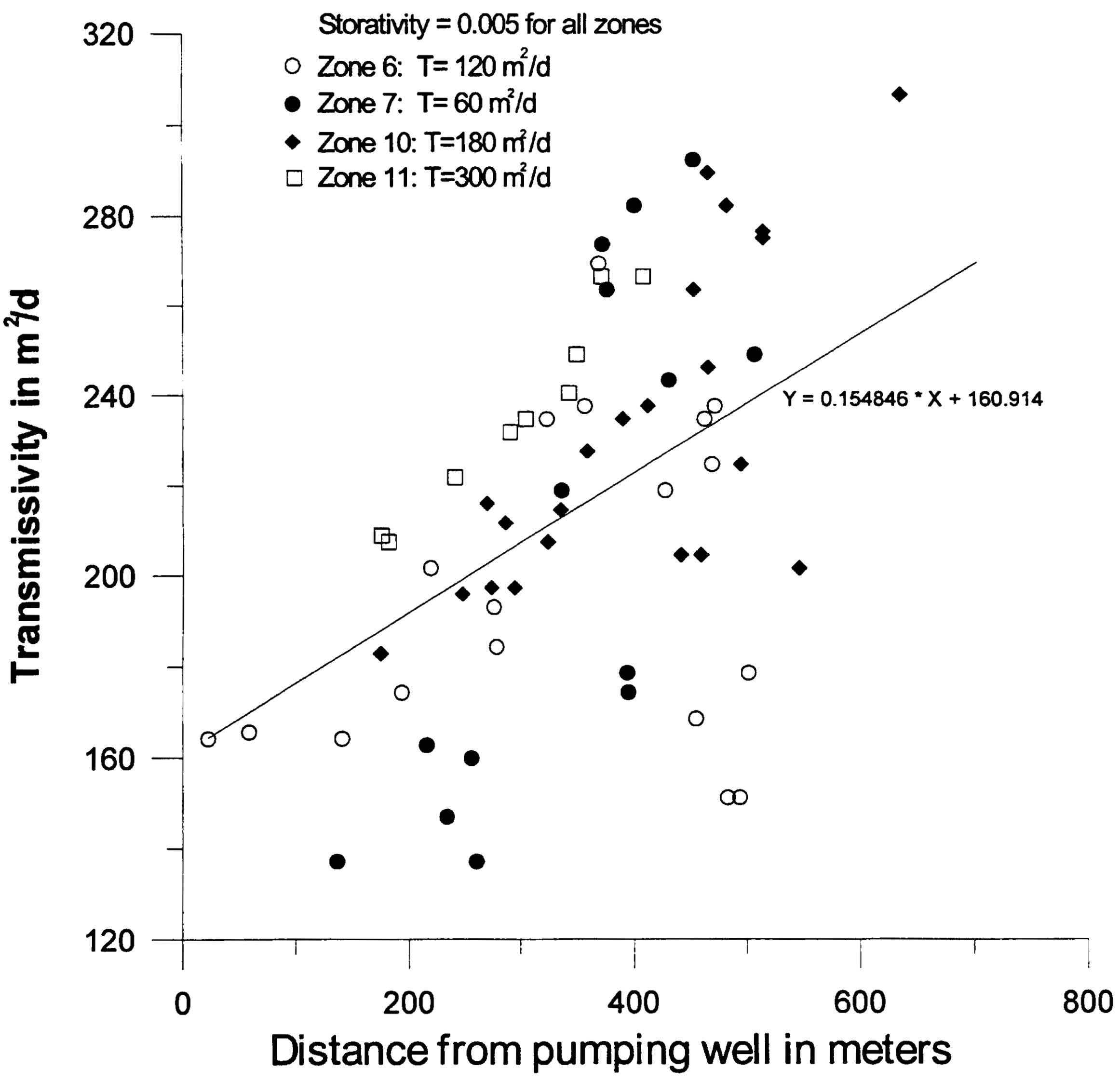

Figure 9. Plot of calculated transmissivity values for the 62 observation wells versus distance from the pumping well for aquifer test 2.

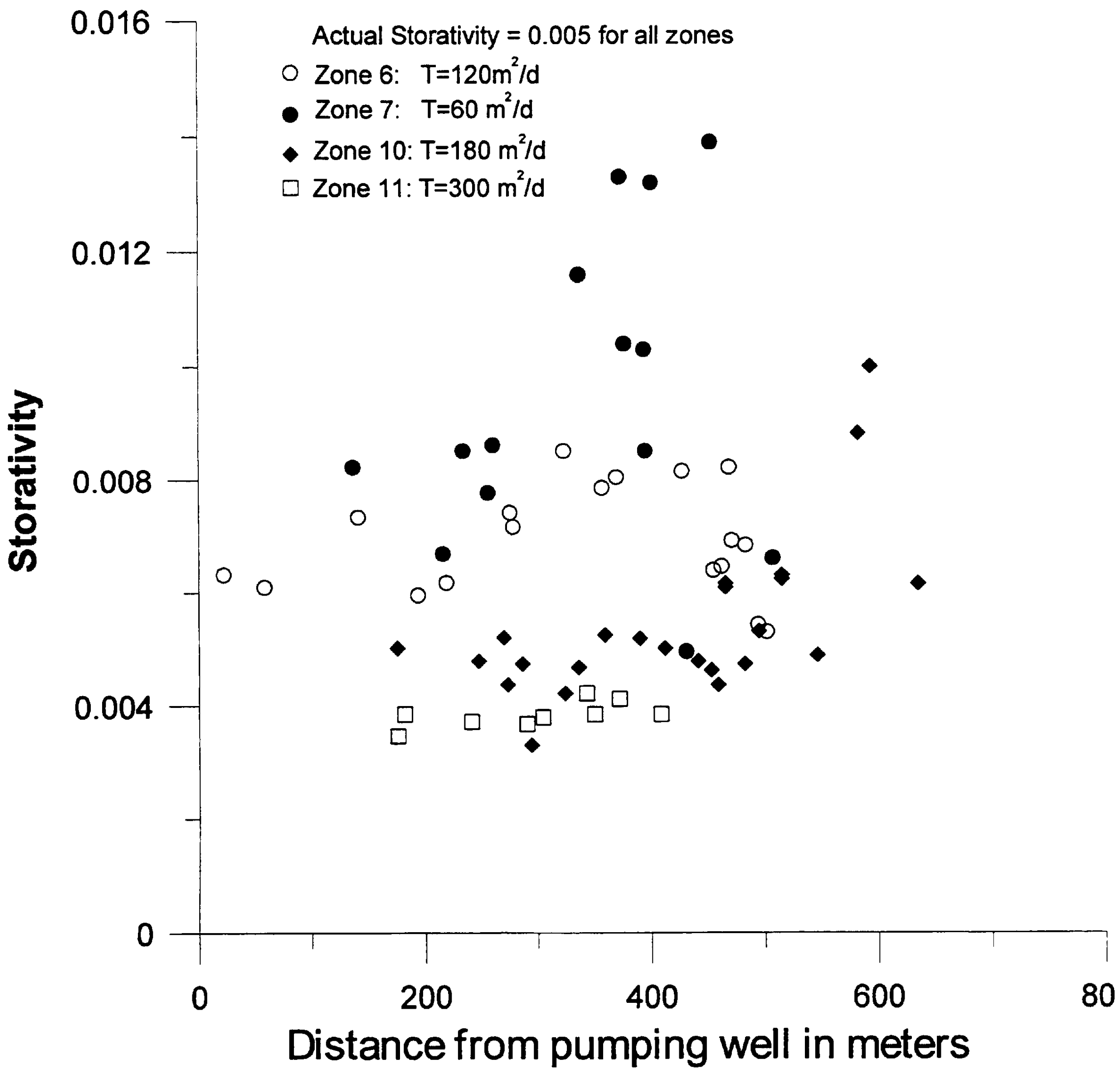

Figure 10. Plot of calculated storativity values for the 62 observation wells versus distance from the pumping well for aquifer test 2.

SUMMARY

Simulation of aquifer tests 1 and 2 provided heterogeneous drawdown data that reflected the net effects of water removed from storage in the aquifer combined with spatially and temporally varying hydraulic gradients. Analysis of these drawdown data by the Theis (1935) method provided indeterminate T and S values rather than unique values because drawdown was not radially symmetrical about the pumping well. T and S values derived from independent drawdown curves generally are not representative of average conditions unless the drawdown curves are defined for large values of t/r^2. T and S values derived from drawdown curves defined only for small values of t/r^2 appear to increase with distance from the pumping well due to poor vertical control provided for the curve matching process. Analysis of drawdown curves as a family provides the vertical control needed to constrain the curve matching procedure. When discrete drawdown curves are analyzed as integral members of a family of curves, better estimates of average T are derived than when the same drawdown curves are analyzed independently.

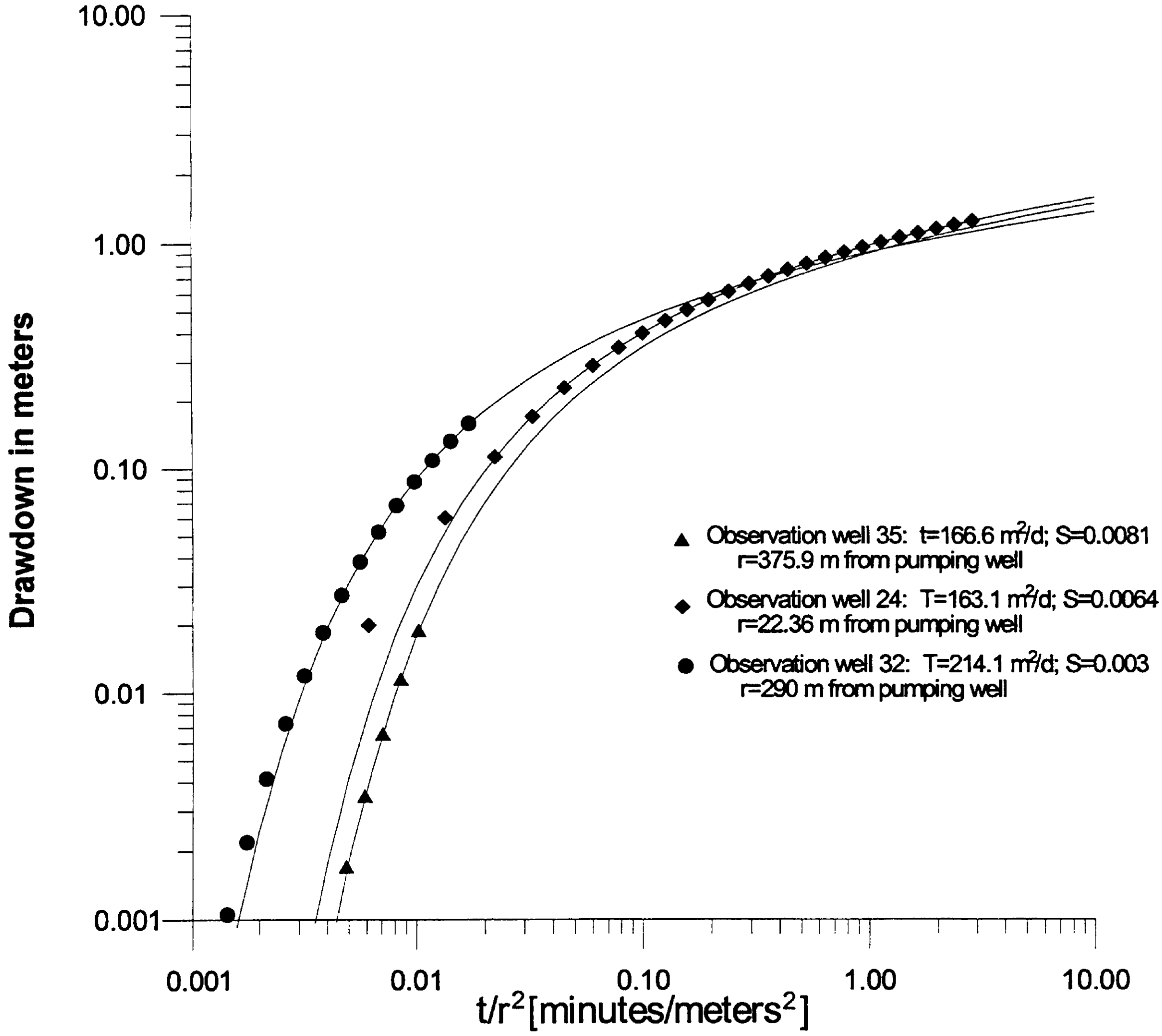

Figure 11. Representative, independent Theis type curve matches for three of the 68 observation wells for aquifer test 2. Theis curve matches were produced by AQTESOLV using nonlinear weighted least-squares parameter estimation.

CONCLUSIONS

Independent analyses of drawdown curves for observation wells located at large radial distances from the pumping well will provide T and S values of uncertain meaning unless the test is of long enough duration (i.e., large values of t/r^2) to allow convergence on a single curve. The results presented here are consistent with the findings of several previous investigators beginning with Toth (1966) that suggested analysis of late-time drawdown data will yield fairly consistent estimates of "apparent" or average T for the volume of aquifer stressed during an aquifer test.

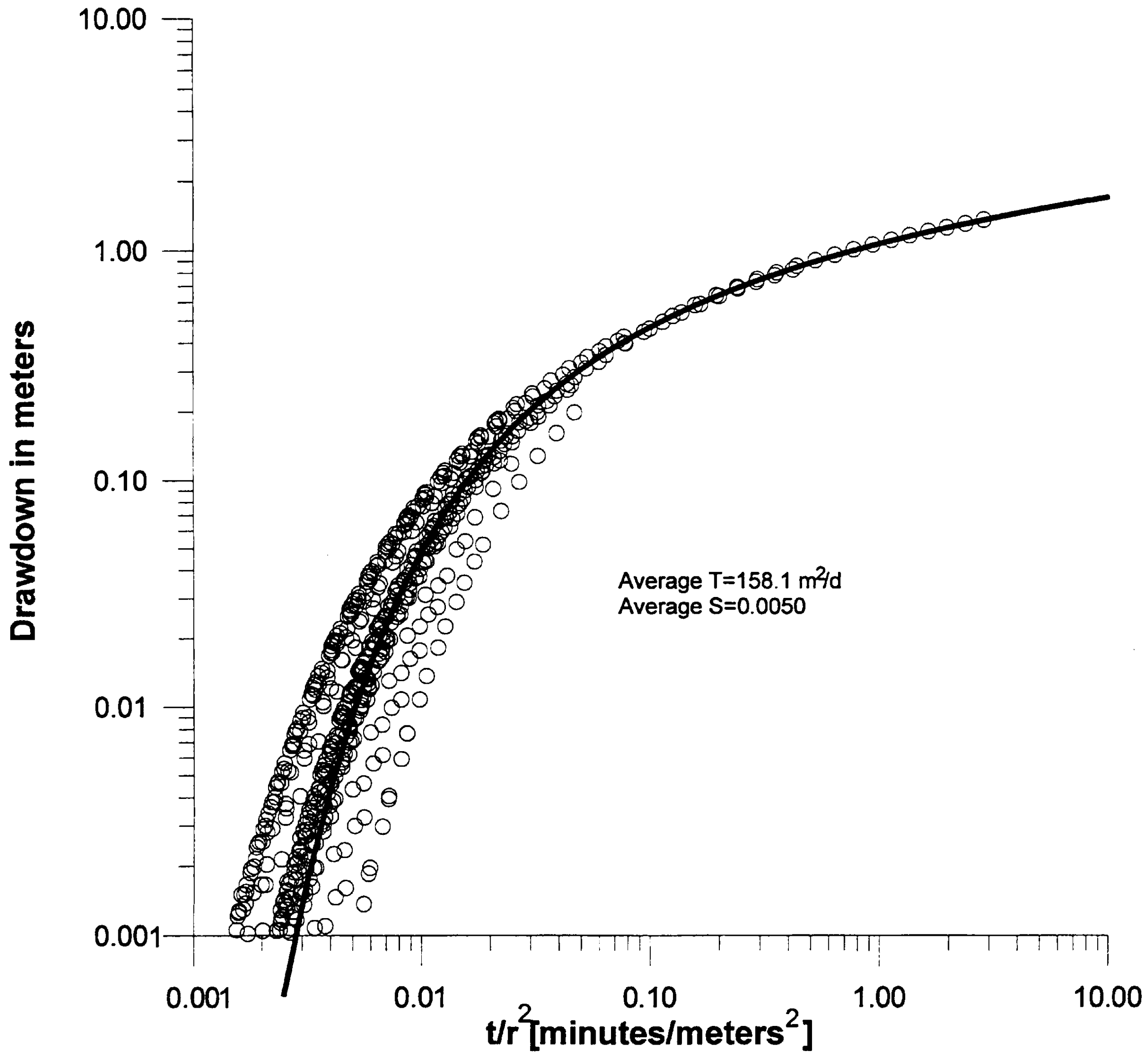

Figure 12. Log-log plot of drawdown versus t/r^2 for all 68 observation wells in aquifer test 1. The Theis curve match to all of the data points was produced by AQTESOLV using nonlinear weighted least-squares parameter estimation.

Early-time drawdown data yield variable T values because of inadequate vertical control during the curve matching procedure. An artificial scale effect develops when multiple drawdown curves, defined for variable periods of time, are analyzed independently. The scale effect develops when steep drawdown curves are matched to the steep portion of the type curve without sufficient vertical control provided by late-time data. Early-time drawdown data are strongly affected by pathline refraction. These conditions allow lateral shifts in the plotted positions of individual drawdown curves relative to the type curve so that T values derived from independent type curve matches appear to increase with distance from the pumping well. Reliable "average" values for T cannot be derived unless the curve matching procedure is constrained by late-time data. This is consistent with the conclusion of Meier et al. (1998) that analysis of observation well drawdown data by the Cooper and Jacob (1946) method yields good approximations of effective T when constrained to late-time data. "Average" values for T can be derived if discrete drawdown curves are analyzed as integral members of a family of curves that includes late-time drawdown data for at least one well. In this case, much tighter ranges of "average" T values are derived for the region within the cone of depression because lateral and vertical shifting of the drawdown data during the curve matching procedure is constrained. Reasonable estimates for T are derived even for distant observation wells with no artificial scale effect. However, the type of averaging that occurs between the pumping well and individual observation wells cannot be defined with certainty.

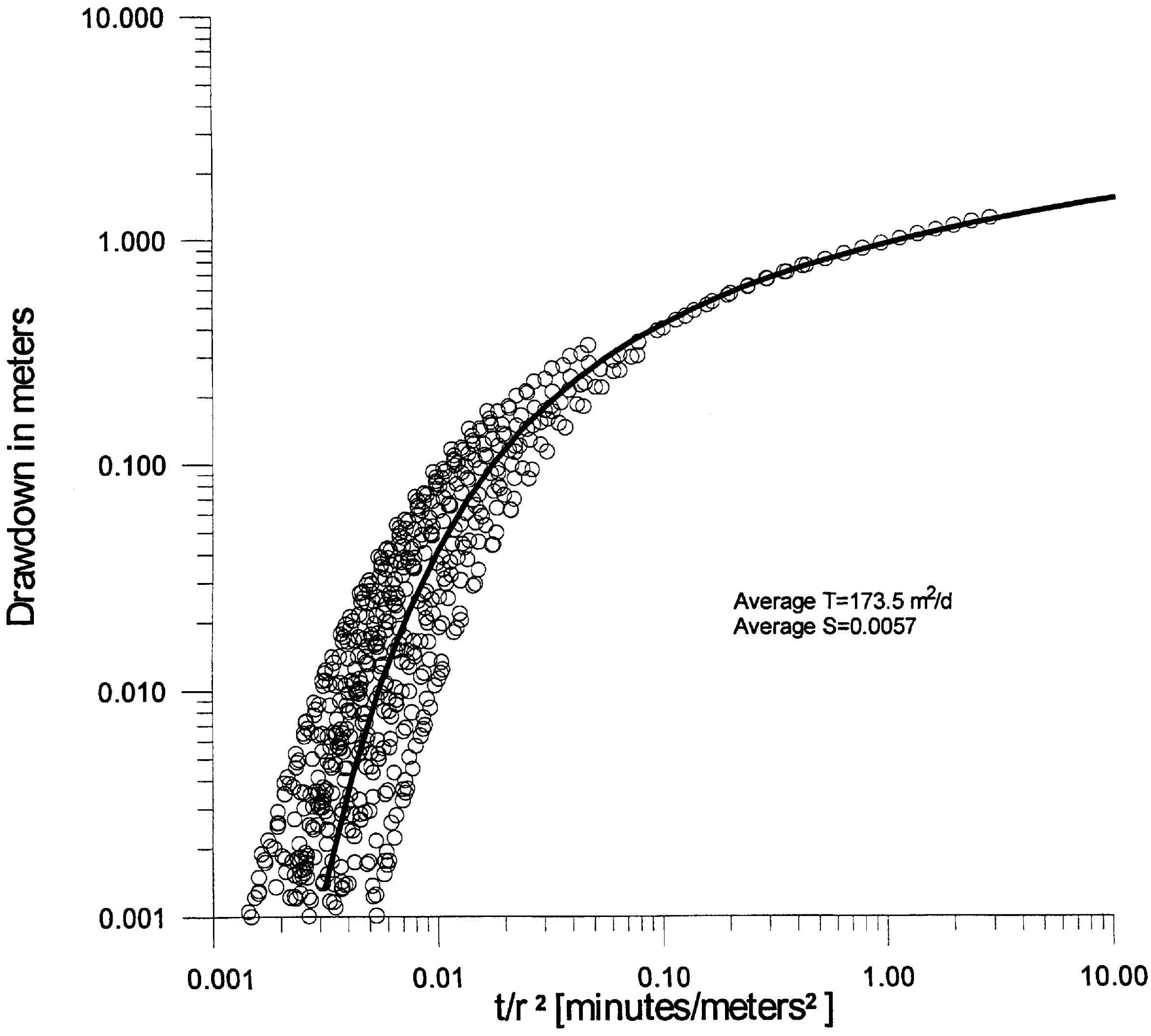

Figure 13. Log-log plot of drawdown versus t/r2 for all 62 observation wells in aquifer test 2. The Theis curve match to all of the data points was produced by AQTESOLV using nonlinear weighted least-squares parameter estimation.

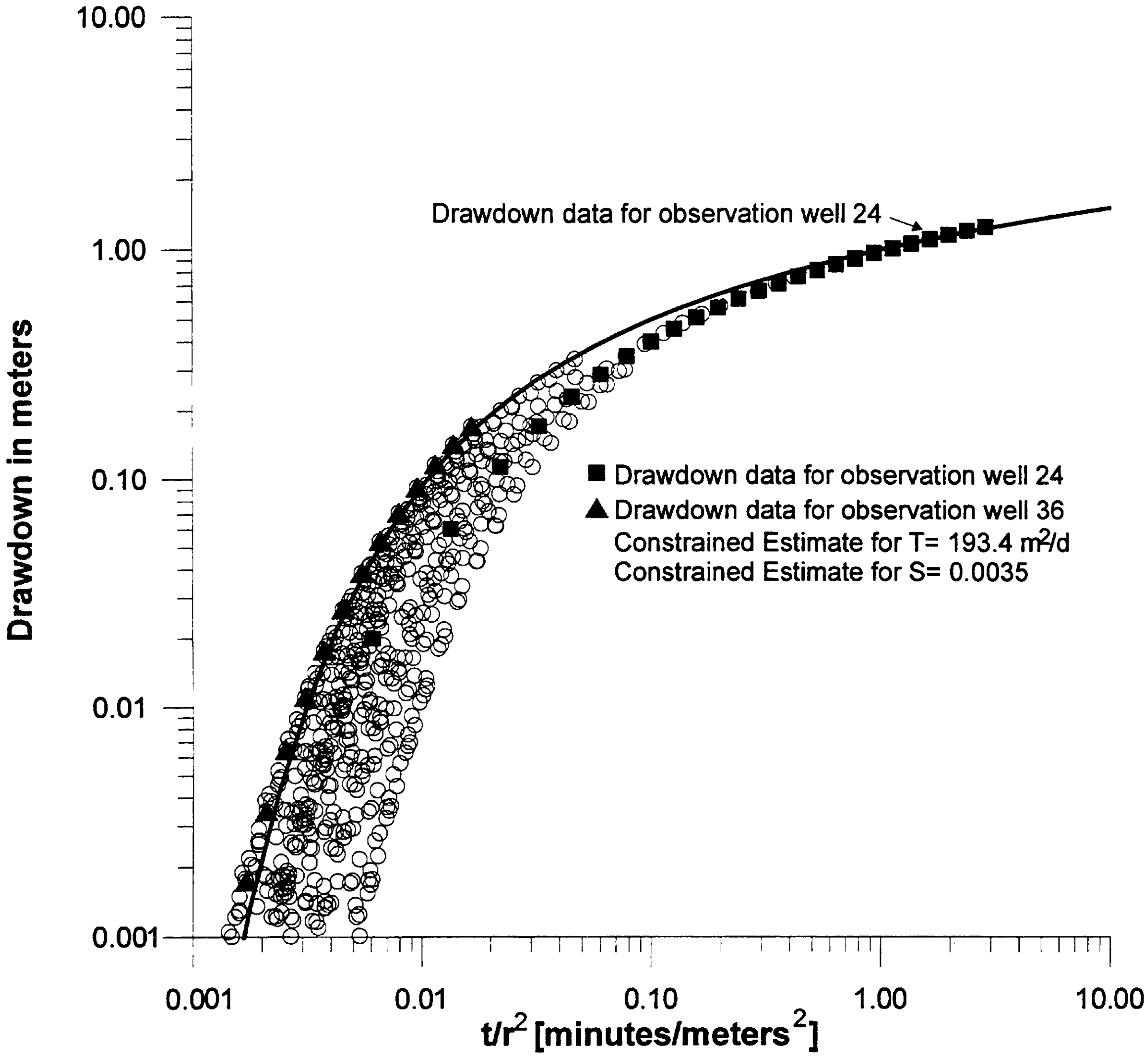

Figure 14. Family of drawdown curves versus t/r^2 for all 62 observation wells in aquifer test 2. The constrained Theis type curve match to the drawdown data for observation well 36 located 371.08 meters from the pumping well is shown. The type curve match is constrained by large values for t/r^2 defined for observation well 24 located 22.36 meters from the pumping well.

Accurate estimates for S cannot be derived from drawdown curves that develop in heterogeneous environments. Vertical and lateral shifting of the drawdown curve relative to the type curve is necessary during the curve matching procedure. The simulations presented in this paper show that estimates for S were affected by the vertical and horizontal positions of the plotted drawdown curves. These plotted positions reflected spatial distortions in the cone of depression that resulted from variations in T when actual S was constant throughout the aquifer.

Separation between the drawdown curves will still materialize for the conditions of high T and low S if the aquifer is heterogeneous and drawdown data are available for both close observation wells and distant observation wells. For these conditions, the early-time drawdown data for the closest observation well will help constrain lateral shifting of the data relative to the type curve, and will provide better estimates of S than could be derived by independent analysis of drawdown data matched to the flat portion of the type curve.

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